An Evolutionary Multi-player Game Model for Two-hop Routing in Delay Tolerant Networks

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***Abstract*—Delay-tolerant networks (DTN) are sparse mobile ad hoc networks where contemporaneous end-to-end path is typically not available. Therefore, nodes act as relays for each other to enable data delivery. The cooperation among mobile nodes however can be hindered by selfish users. Incentive schemes are inevitably introduced to regulate the behavior of DTN users. The motivation of this paper is to seek conditions under which cooperators can survive, and even prevail in DTN without incentives. We study the formation of cooperation in DTN routing following an Evolutionary Game Theory (EGT) approach. In particular, the two-hop routing protocol is assumed to be adopted which applies to a large class of DTN routing schemes. We first formulate the two-hop DTN routing as a multi-player game. Using the concept of Evolutionarily Stable Strategy (ESS), we show that defection always dominates cooperation when the population is infinite and well-mixed. Recent developments in evolutionary biology reveal that in finite and structured populations such as graphs, cooperation is nevertheless promoted. We derive a sufficient condition for cooperators to be favored over defectors on K-regular graph. Simulation results validate our theoretical finding, and prove that cooperation indeed can prevail in DTN routing games without incentives.**

I. Introduction

In Delay Tolerant Networks (DTN), end-to-end connectivity is frequently disrupted due to node mobility or power limitations. In order to overcome the intermittent connectivity, mobile nodes adopt a store-and-forward scheme: they forward/replicate the message to other nodes upon encounter until one of the copies reaches the destination. Therefore the message delivery efficiency depends on how the source node chooses relays and the number of message copies in network. There have been many routing schemes developed to optimize DTN message forwarding [12], [19], [28], [31], [34], among which the multi-copy two-hop routing scheme strikes a good balance between performance, overhead, and complexity. The problem, however, is that two-hop routing relies on cooperations between neighbors, and the system can be easily damaged in the presence of selfish users. Selfish nodes may exploit resource of the neighbors and preserve their own by dropping the messages they are supposed to relay.

To stimulate cooperation in DTN, credit-based and reputation-based incentive schemes have been developed [26]. The idea is to reward cooperative actions by virtual payments or by raising users' reputations. Selfish users are isolated either due to the lack of currency to purchase relays or bad reputations. However, deploying these mechanisms inevitably incurs additional costs. In credit-based system, management of transactions and tamper-proof hardware need to be implemented, while in reputation-based system a network-wide reputation monitoring and update must take place.

In contrast to the above cooperation enforcement schemes, Evolutionary Game Theory (EGT) [32] aims to study the spontaneous strategy selection of users without external forces. It captures the dynamic change of network state by evolving user strategies over time based on natural selection. Interactions of mobile users (players) in DTN can be modeled as social dilemma games. Individual obtains payoff by participating in the game with its neighbors. Players update their strategies by comparing their payoff with the neighbors'. In infinite and well-mixed population, the predominant strategy reaching equilibrium state is an Evolutionarily Stable Strategy (ESS), a refinement of Nash equilibrium in the EGT setting.

In this paper, we study the cooperation issue in two-hop DTN routing using the EGT framework. We formulate the two-hop DTN routing as a multi-player game, or DTN Multi-player Game (DMG) for short. Two strategies can be adopted by a player: cooperation (C) or defection (D). In any mixed population, the average payoff of defectors is always higher than the cooperators, i.e. defection dominates cooperation. Therefore, individual has no incentive to cooperate. Indeed, we obtain that the ESS in DMG is defection. This result, however, is restricted to networks of *infinite number of users* and *well-mixed interactions*. The two assumptions can be challenged by the following facts: *1)* some well-known DTN datasets and targeted scenarios contain only tens to hundreds of nodes [18], [29]; *2)* interactions in DTN are correlated rather than random as in the well-mixed case [16]. Therefore, the result may not be able to accurately approximate the outcome of real-world DTN. A more authentic setting would be DMG over finite and structured populations, where interactions are confined within a group of similar entities.

There has been much interest in studying evolutionary games in structured populations [1], [5]. Recent developments in [22], [23] revealed that cooperation can be promoted when the evolution takes place on lattice or random graphs. The condition for cooperators to dominate defectors is given by the "σ-rule", which depends not only on the payoffs but also on the underlying structures. An analytical approximation of the "σ-rule" is derived in [23] for regular graph of arbitrary degree. Motivated by this line of research, we make the following contributions:

– We formulate the two-hop DTN routing as an evolutionary multi-player game termed DMG. The payoff function is equal to the successful message delivery rate with the help of cooperative neighbors, minus the total cost to help neighbors.
– Observing that the ESS of DMG in infinite and well-mixed population is defection, we turn our attention to DMG over structured populations. A sufficient condition is derived under which cooperation is favored over defection in DMG on regular graphs with arbitrary degree K. Specifically, cooperation dominates when the cost to help a neighbor scales as $O(\frac{1}{K^2})$ for large K.
– The theoretical result is validated through numerical simulations on three types of graphs: ring, random regular graph, and community-based graph. Assumptions and implications of simulation results are discussed.

Our result substantially differs from previous work [9], [30] by showing that under the assumption of natural selection and structured populations, cooperation can be favored by the system even without external incentive schemes. The rest of this paper is structured as follows. Section II summarizes related work. Section III introduces the network and game models. In Section IV, we derive the main result of this paper on K-regular graph. This result is verified in Section V. Finally, conclusion is drawn in Section VI.

II. Related Work

A. Delay Tolerant Networks (DTN)

Data routing in DTN is challenging as the network is sparse and there is typically no end-to-end path for message transmission. Therefore, much attention has been paid to the development of efficient routing algorithm in DTN. The first attempt is Epidemic routing [31], in which source replicates packets whenever it meets an intermediate relay. This scheme approaches the optimal delivery ratio and delay by flooding the entire network. To reduce the overhead, routing protocols that limit the number of message copies at source nodes are proposed. Spray-and-Wait [28] is a well-known protocol in this category. In the spray phase, source node spreads all its message copies to the first few encountered nodes. In the wait phase, nodes who have a copy of the message deliver the message directly to the destination. The optimal performance and resource consumption trade-off is also extensively studied in [2], [4], [8], [19], [33], [34] etc. Most theoretical work is targeting at the two-hop routing case, as it simplifies the route selection and highlights the importance of choosing the right relay nodes.

Another branch of DTN research focuses on incentive schemes for DTN routing protocol. DTN networks are ad-hoc networks comprising human users. Therefore users are assumed to be selfish and may exploit other people's resource. Incentive schemes in DTN fall into three categories: 1) credit-based incentive schemes which introduce virtual currency to regulate message forwarding in DTN; 2) reputation-based schemes which evaluate the cooperation levels of users, and selfish users get less service than cooperative users; 3) tit-for-tat (TFT) schemes where users contribute equal amount of resource as they obtain from other users. An informative review of these incentive schemes can be found in [26]. The challenge, however, lies in the cost of operating and securing the incentive systems. Some researchers are seeking conditions for mobile users to behave cooperatively without external incentive schemes [9], [30].

B. Evolutionary Game Theory (EGT)

EGT extends the concepts of game theory to evolving populations [32]. It models an evolutionary (Darwinian) process where the more fit individuals pass on their strategies to more offspring and increase their representation in the population. Lots of effort is devoted to studying the *replicator equations* of evolutionary games, which characterize the strategy dynamics under the influence of natural selection [14]. The attractors (fixed points) of the equations are called Evolutionarily Stable Strategies (ESS), a refinement of Nash Equilibrium. A strategy is ESS if it cannot be invaded by any alternative strategies. The EGT model serves as a mathematical framework to understand cooperative behaviors in a wide range of biological and social games [20]. Early works typically assume pair-wise interactions in infinite and complete mixing populations.

More recently, population structures such as lattice and complex networks are introduced to the EGT model [21], [25]. While the impact of spatially structured populations on evolutionary dynamics is mostly studied with the aid of computer simulations, a surprisingly simple analytical result, known as the "σ-rule" [22], reveals that structured populations promote the evolution of cooperation. Multi-player games representing group interactions can be naturally embedded in the structured populations and extend two-player games to a generic form [10]. The σ-rule for multi-player games is developed in [23], [24].

The EGT methodology has been applied to many networking research problems. To name a few, [3] formulated cooperative streaming as an evolutionary game and derives the ESS strategy of the game; [17] proposed an EGT framework to model the information diffusion dynamics in social networks; [6], [27] studied the cooperation in Public Goods Game under topological changes in vehicular and mobile social networks. EGT model for DTN routing protocol also exists in literature [7]. It integrated the EGT model with credit-based incentive to motivate cooperation in packet forwarding games. The difference is that [7] assumes infinite and well-mixed populations, and its main focus is mechanism control with resource constraint. In this paper, we consider finite populations on graph structures and concentrate on the evolution dynamics that is approximated by the σ-rule.

III. Network and Game Models

A. DTN Network Model

We consider a DTN network of N mobile users. The DTN network is sparse and users are disconnected most of the time in their movement. Mobile users communicate only when they are in proximity to each other. We refer to such opportunity of communication between two users as *contact*. The inter-contact times (ICT) between neighboring nodes are independently and identically distributed (iid) exponential variables with parameter $\lambda > 0$. This assumption has been justified in previous work [13].

In the network, every user is a source that constantly generates messages destined to some destination. Due to the intermittent connectivity and a lack of instantaneous end-to-end paths, messages are transmitted in a store-and-forward manner. In this paper, we assume that a two-hop routing scheme is employed. The source either directly delivers the message to the destination, or replicates the message to an intermediate relay. The relay will deliver the message when it meets the destination. Therefore every message reaches the destination in at most 2 hops. We assume that each message is relevant for a time interval of length at most τ. For timely delivery, the source seeks help from as many relays as possible. When the number of relays is equal to k, the delivery time is

$$T = \min\{T_{sd}, T_{sr_1} + T_{r_1 d}, \ldots, T_{sr_k} + T_{r_k d}\}$$

where T_{sd} is the time for direct delivery, and $T_{sr_k} + T_{r_k d}$ is the time for delivery via relay k. Based on the assumption of iid exponential ICT, the probability of timely delivery is

$$\begin{aligned}\mathbb{P}_k(T < \tau) &= 1 - \mathbb{P}(T_{sd} \geq \tau) \prod_{i=1}^{k} \mathbb{P}(T_{sr_i} + T_{r_i d} \geq \tau) \\ &= 1 - e^{-\lambda\tau} \left(\frac{1+\lambda\tau}{e^{\lambda\tau}}\right)^k\end{aligned} \tag{1}$$

where we use the fact that $T_{sr_i} + T_{r_i d}$ is $Erlang(2)$, and $\mathbb{P}(T_{sr_i} + T_{r_i d} \geq \tau) = \frac{1+\lambda\tau}{e^{\lambda\tau}}$.

B. Evolutionary Game Theory Model

We adopt the following standard EGT settings:

– The population is formed by the N mobile users.
– Every user chooses a pure strategy from the strategy set $\mathcal{S} = \{C, D\}$, where C stands for cooperation and D means to defect.
– Let p_C be the fraction of users taking strategy C. Since there are only two strategies in $\mathcal{S}$, p_C determines the strategy distribution across the population (as $p_D = 1 - p_C$). Therefore we use p_C to denote the current state of the population.
– The N users obtain payoff from the interactions with neighbors. Individual payoff depends on its own strategy as well as the strategy of neighbors. The payoff of a user i playing strategy $s_i \in \mathcal{S}$ is denoted by $f_i(s_i, p_C)$, and the average payoff in a population with state p_C is $F(p_C, p_C) = p_C f_i(C, p_C) + (1 - p_C) f_i(D, p_C)$.
– Every player updates strategy based on the payoff obtained from the current interaction. This update in turn changes the population state, which will be carried on to the next round of interaction. The evolution process repeats for many times until the state reaches equilibrium.

With this EGT setting, we can formulate the two-hop DTN routing problem as a multi-player game (DMG). The payoff of an individual user depends on the group of users it interacts with, which we call its neighborhood. For user i whose neighborhood is N_i, among which k_i are cooperators, the payoff is defined as

$$\begin{aligned}f_{i|N_i,k_i}(D) &= \gamma_i \mathbb{P}_{k_i}(T < \tau) \\ f_{i|N_i,k_i}(C) &= f_{i|N_i,k_i}(D) - \sum_{j \in N_i} \gamma_j \varepsilon\end{aligned} \tag{2}$$

We assume that users playing C always relay packets for all neighbors. Here γ_i is the message rate at source i, and ε represents the cost (e.g. energy consumption, bandwidth, memory usage, etc.) for relaying one message. The defined payoff for cooperators equals to the benefit obtained from relays minus the contribution to others. We assume homogeneous packet rate in the population $\gamma_i = 1$. An implicit assumption made here is that the k_i cooperative neighbors can all reach the destination of user i. While this assumption is acceptable in many population structures, it might not be true when users are randomly connected. We will revisit this issue in Section V-C.

We first consider a well-mixed DTN network with infinite population, where everyone contacts $N-1$ random users with equal frequency λ in each round. The expected individual payoff can be written as

$$\begin{aligned}f_i(D, p_C) &= \sum_{k=0}^{N-1} \mathbb{P}(k_i = k) f_{i|N_i,k_i}(D) \\ &= \sum_{k=0}^{N-1} \binom{N-1}{k} p_C^k (1 - p_C)^{N-1-k} \mathbb{P}_k(T < \tau) \\ f_i(C, p_C) &= f_i(D, p_C) - (N-1)\varepsilon\end{aligned}$$

We are interested in the fraction of cooperators p_C when the system is at equilibrium state. The equilibrium point of EGT is given by the Evolutionarily Stable Strategy (ESS). More formally, we have the following definition:

Definition 1. *A population state p_C^* is an ESS if for any mutant strategy $mut \neq p_C^*$, there exists some $\epsilon_M \in (0,1)$, which may depend on mut, such that $\forall \epsilon \in (0, \epsilon_M)$ the following holds,*

$$F(p_C^*, \epsilon \cdot mut + (1-\epsilon) p_C^*) > F(mut, \epsilon \cdot mut + (1-\epsilon) p_C^*) \tag{3}$$

Due to the fact that $f_i(D, p_C) > f_i(C, p_C), \forall p_C$ when $\varepsilon > 0$, we immediately come to the conclusion that $p_C^* = 0$ is an ESS to DMG. This is not surprising as the game under study is the N-player Prisoner's Dilemma with non-linear payoff function, whose Nash Equilibrium is defection. This pessimistic result implies that incentive scheme has to be introduced by the network operator in order to stimulate co-operation. Previous papers have come to the same conclusion [9], [30].

It is worth noting that the above ESS applies to a well-mixed population. In reality, the mobility patterns in DTN have strong location preference [11], [15]. Therefore, the neighborhood of each user is restricted to a small set of $friend$ nodes. Interactions with these friends can be aggregated into a contact graph. The evolutionary dynamics on

Table I: Payoff of DMG on K-regular Graph

Number of C neighbors k	0	1	...	K
$f_{K,k}(C)$[1]	$1-e^{-\lambda\tau}-K\varepsilon$	$1-e^{-\lambda\tau}\left(\frac{1+\lambda\tau}{e^{\lambda\tau}}\right)-K\varepsilon$	...	$1-e^{-\lambda\tau}\left(\frac{1+\lambda\tau}{e^{\lambda\tau}}\right)^K-K\varepsilon$
$f_{K,k}(D)$	$1-e^{-\lambda\tau}$	$1-e^{-\lambda\tau}\left(\frac{1+\lambda\tau}{e^{\lambda\tau}}\right)$	...	$1-e^{-\lambda\tau}\left(\frac{1+\lambda\tau}{e^{\lambda\tau}}\right)^K$

graphs has attracted lots of attentions. Recent progress in [23] revealed the condition for cooperation to be favored on K-regular graph. Unlike in infinite and well-mixed populations, preference for cooperation or defection on graph structures is measured using the metric *fixation probability*. The fixation probability of cooperation, denoted by ρ_C, is the probability that a single cooperator out of $N-1$ defectors turns the whole population into cooperators, i.e. the probability of convergence to complete cooperation. Similarly, the fixation probability of defection, denoted by ρ_D, is the probability of convergence to complete defection from a single defector. If $\rho_C > \rho_D$, then the long-term frequency of cooperators is higher than the frequency of defectors [5], or equivalently, C is favored over D by nature selection. [23] analytically showed the condition for $\rho_C > \rho_D$.

IV. Evolution of DMG on K-regular Graph

Now we focus on the evolution of DMG on K-regular graph. Each user has exactly K neighbors, and plays a single $(K+1)$-player game per round. The payoff obtained from the $(K+1)$-player game is shown in Table I. Next, we follow closely the methodology in [23] to study the fixation probability ρ_C and ρ_D.

After users obtain their payoffs from $(K+1)$-player DMG, the population state is updated by a Moran death-birth (DB) process. Specifically, a random individual i is chosen to die and its K neighbors compete to take its place. The probability that a particular neighbor j wins the competition is proportional to its fitness, defined as $\mathcal{F}_j = 1-\omega+\omega\times f_{K,k_j}(s_j)$ where $\omega \ll 1$. Therefore, the probability that a cooperator wins is $\sum_{j\in K_i^C}\mathcal{F}_j / \sum_{j\in K_i}\mathcal{F}_j$, where K_i is the set of neighbors of i and K_i^C is the set of cooperative neighbors. The Moran DB process stochastically updates the population state.

From [23], the condition for $\rho_C > \rho_D$ in DMG on K-regular graph is given by the following σ-rule:

$$\sum_{k=0}^{K}\sigma_k d_k > 0 \tag{4}$$

where $\sigma_0, ..., \sigma_K$ are the normalized structure coefficients characterizing the underlying population structure, and $d_k = f_{K,k}(C) - f_{K,K-k}(D), \forall k = 0,1,...,K$.

The structure coefficients of different graphs have been derived. Here we concentrate on two cases: $(N-1)$-regular graph, which corresponds to a well-mixed population of finite size N, and the general K-regular graph where $K \ll N$. We will see how the structure coefficients promote cooperation in K-regular graph compared to the well-mixed case. A sufficient condition under which cooperation is favored over defection in DMG on K-regular graph is then derived.

A. The σ-rule in Well-mixed Population

The structure coefficients in well-mixed population is given by the following lemma.

Lemma 1. *[10] For regular graph of degree $K = N-1$, the structure coefficients are*

$$\sigma_k^{wm} = \begin{cases} \frac{1}{N-1}, & 0 \le k < N-1 \\ 0, & k = N-1 \end{cases} \tag{5}$$

The above σ-rule has the same implication as ESS in well-mixed population, as reflected in the following theorem.

Theorem 2. *In a finite and well-mixed population of size N, where individual plays the N-player DMG with everyone else, $\rho_C < \rho_D$ always holds.*

Proof. Plugging equation (5) into condition (4) we have

$$\begin{aligned}\sum_{k=0}^{N-1}\sigma_k^{wm}d_k &= \frac{1}{N-1}(\sum_{k=0}^{N-2}f_{N-1,k}(C) - \sum_{k=1}^{N-1}f_{N-1,k}(D))\\ &= \frac{1}{N-1}(f_{N-1,0}(D) - f_{N-1,N-1}(D)) - (N-1)\varepsilon\\ &= \frac{1}{(N-1)e^{\lambda\tau}}(\left(\frac{1+\lambda\tau}{e^{\lambda\tau}}\right)^{N-1} - 1) - (N-1)\varepsilon\\ &< 0.\end{aligned}$$

which indicates that $\rho_C < \rho_D$. □

Remark 1. *Compared to Theorem 2, the ESS of $p_C^* = 0$ we obtain in section III-B applies to an infinite and well-mixed population where everyone is involved in DMG with N random neighbors per round. However, in both cases defection is favored over cooperation. This is because the well-mixed structure facilitates the invasion of defectors to cooperators [25]. Cooperators are easily exploited when facing defectors.*

B. The σ-rule on K-regular Graph

We have seen that the σ-rule does not favor cooperation in well-mixed population. In contrast, both [23] and [22] show that the σ-rule on K-regular graph favors cooperation when $K \ll N$. We introduce the following lemma from [23].

Lemma 3. *[23] For regular graph of degree $K \ge 3$ and $K \ll N$, the structure coefficients can be approximated by*

$$\sigma_k = \frac{(K-2)^{K-1-k}}{(K+2)(K+1)K^2}\sum_{l=0}^{K-1}(K-l)\{[K^2-(K-2)l]\upsilon_{l,k,K} + [2K+(K-2)l]\varsigma_{l,k,K}\} \tag{6}$$

[1]As payoff on K-regular graph is symmetric, we drop the i index from the payoff notation $f_{i|N_i,k_i}$.

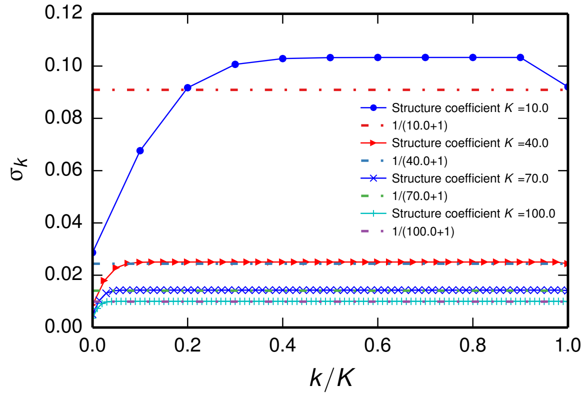


Figure 1: Structure coefficients on K-regular graph.

$\forall 0 \le k \le K$, *where*

$$\upsilon_{l,k,K} = \binom{K-1-l}{K-1-k}\frac{1}{(K-1)^{K-1-l}} + \binom{l}{K-k}\frac{K-2}{(K-1)^{l}},$$

$$\varsigma_{l,k,K} = \binom{K-1-l}{K-k}\frac{K-2}{(K-1)^{K-1-l}} + \binom{l}{K-1-k}\frac{1}{(K-1)^{l}}.$$

Due to the complex form of equation (6), we plot the structure coefficients of K-regular graph for different values of K. As shown in Fig. 1, the structure coefficients σ_k asymptotically approach $1/(K+1)$ when $K \gg 1$ and $k \gg 1$. In the meantime, σ_0 and σ_1 are always smaller than $1/(K+1)$. This quasi-increasing property clearly distinguishes K-regular graph from well-mixed graph, whose structure coefficient is given in equation (5)[2]. The concept that characterizes this difference is the *containment order* [24]. Population structure $\mathcal{P}_1$ is greater than population structure $\mathcal{P}_2$ in containment order, if the set of games for which cooperation is favored under $\mathcal{P}_2$ is contained in the set of games for which cooperation is favored under $\mathcal{P}_1$. A sufficient condition for $\mathcal{P}_1$ to be greater than $\mathcal{P}_2$ in containment order is that the difference in structure coefficients under $\mathcal{P}_1$ and $\mathcal{P}_2$, $\sigma_k^1 - \sigma_k^2, \quad 0 \le k \le K$, has exactly one sign change from $-$ to $+$. Using this property, we can easily obtain that K-regular graph is greater than the well-mixed population in containment order. Therefore, cooperation evolves more easily in K-regular graph than in the well-mixed population.

C. Evolution of DMG on K-regular Graph

Next, we study the condition under which cooperators are favored on K-regular graph. The only assumption we make on the structure coefficients is the quasi-increasing property: $\sigma_k - \sigma_{K-k} \ge 0$ when $k \ge K^* \triangleq \lfloor \frac{K}{2} \rfloor$, which is obvious from Fig. 1. This quasi-increasing property on K-regular graph results its higher containment order. Therefore, condition (4) has a chance to be satisfied when DMG evolves on a K-regular graph. In the following, we present the main result of this paper: a sufficient condition for cooperation to be favored on K-regular graph. Specifically, based on Lemma 3, we show that $\rho_C - \rho_D > 0$ when the cost ε is sufficiently small.

[2]The general expression for structure coefficients of well-mixed populations with group size equal to $K+1$ can be found in [23].

Theorem 4. *Given parameters $\lambda\tau$ and K, there exists some constant $\epsilon_{\lambda\tau,K} \in (0,1)$ such that the cooperators are favored in DMG on K-regular graph if*

$$\begin{aligned} &\varepsilon < \epsilon_{\lambda\tau,K}\frac{1}{K^2} \sim O(\frac{1}{K^2}), \\ &\epsilon_{\lambda\tau,K} = e^{-\lambda\tau}\{(1-\frac{1}{e})[1-\left(\frac{1+\lambda\tau}{e^{\lambda\tau}}\right)^K] \\ &\quad + (1-\frac{2}{e})[\frac{1+\lambda\tau}{e^{\lambda\tau}} - \left(\frac{1+\lambda\tau}{e^{\lambda\tau}}\right)^{K-1}]\}, \end{aligned} \tag{7}$$

when $N \gg K \gg 1$.

Proof. See Appendix A. □

V. Numerical Simulations

In this section, we verify the theoretical finding with numerical simulations. First, we introduce the simulation setup. Three different population structures are considered: circular, random, and community-based graphs. Evolutionary dynamics of DMG is then simulated over the structured populations. We investigate how the difference in the fixation probability $\rho_C - \rho_D$ changes with respect to relevant parameters. Assumptions, results and implications are discussed as well.

A. Simulation Setup

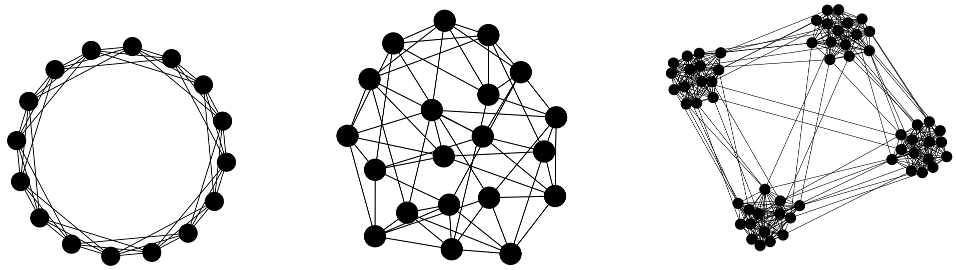
To assess the theoretical approximation in Theorem 4, we implement the evolutionary algorithm of DMG on three different graphs[3]:

- **Ring:** Deterministic graph where nodes are placed in a circle. Every node is connected to $K/2$ nodes to the left, and $K/2$ nodes to the right.
- **Random Regular Graph:** Random graph where every node has the same degree K.
- **Community-based Graph:** Random graph that has high modularity coefficient and can be divided into communities/modules by community detection algorithms. Node has dense connections inside their own community and sparse connections to other communities.

Examples of the graphs are shown in Fig. 2. In ring and random regular graph, every node is connected to exactly K neighbors. In community-based graph, as most edges are connecting nodes that belong to the same community (some nodes may have edges pointing to other random communities), the degree of each node is close to the community size. We keep the size of all communities equal, so that nodes have roughly the same degree $\overline{K} \simeq K$.

Unless otherwise specified, we use the following simulations parameters: $N = 100$, $6 \le K \le 20$, $\lambda\tau = 0.1$ (infrequent contact) or 1 (frequent contact). The fixation probability of cooperators ρ_C is calculated as the fraction of simulation runs in which a single cooperator turned the whole population to cooperators out of 10^6 runs. The fixation probability of defectors ρ_D is calculated similarly. Next, we present simulation results for $\rho_C - \rho_D$ under different structure and parameter conditions.

[3]The graphs are generated using Matlab Tools for Network Analysis. Available: http://strategic.mit.edu/downloads.php?page=matlab_networks

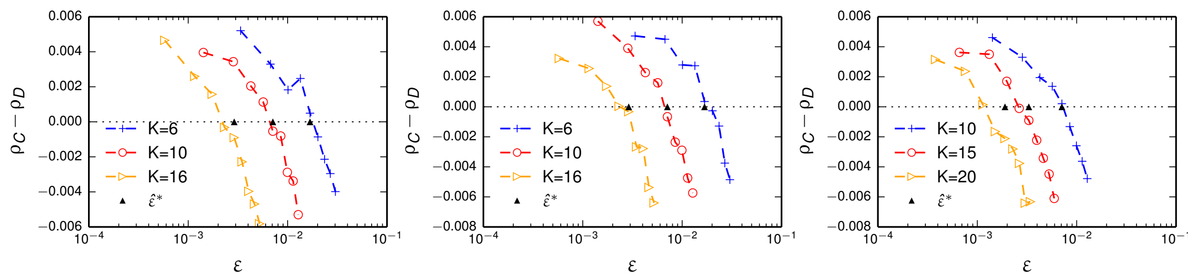


(a) Ring graph.

(b) Random regular graph.

(c) Community-based graph.

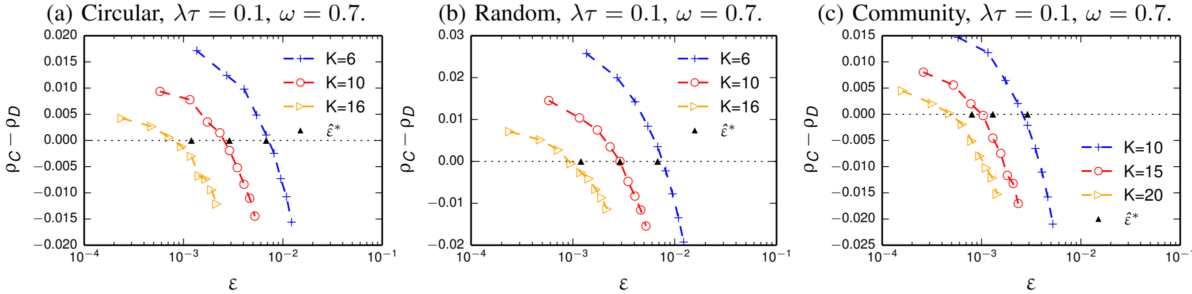
Figure 2: Population structures under study.



(a) Circular, $\lambda\tau = 0.1$, $\omega = 0.7$.

(b) Random, $\lambda\tau = 0.1$, $\omega = 0.7$.

(c) Community, $\lambda\tau = 0.1$, $\omega = 0.7$.



(d) Circular, $\lambda\tau = 1$, $\omega = 0.5$.

(e) Random, $\lambda\tau = 1$, $\omega = 0.5$.

(f) Community, $\lambda\tau = 1$, $\omega = 0.5$.

Figure 3: Simulation results for $\rho_C - \rho_D$.

B. Simulation Results

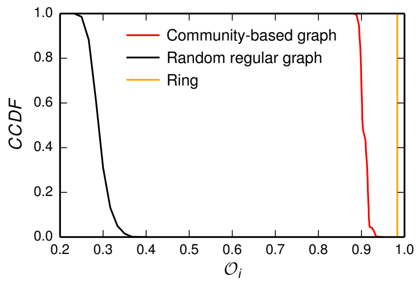
Simulation results in different graphs are shown in Fig. 3. Note that when computing the fitness $\mathcal{F}_i = 1 - \omega + \omega \times f_{K,k_i}(s_i)$, we do not follow the requirement $\omega \ll 1$ as commonly assumed in Moran process. The reason lies in the payoff function of DMG. In common multi-player games such as the N-player prisoner's dilemma or volunteer's dilemma [23], the maximum individual payoff is $Kb - c \simeq K(K-1)c$ where $b \simeq Kc$ is the benefit and $c = 1$ is the cost for cooperators. The payoff becomes much larger than the baseline fitness $1-\omega$ when $K \gg 1$. In our case, the maximum individual payoff $f_{K,K}(C) < f_{K,K}(D) < 1$ always holds. Therefore we adopt $\omega = 0.7$ when $\lambda\tau = 0.1$, and $\omega = 0.5$ when $\lambda\tau = 1$ to make the payoff and baseline fitness $1 - \omega$ comparable.

From Fig. 3, the difference in fixation probabilities decreases with cost ε. Moreover, this difference has a unique zero point at a critical cost ε^*, and $\rho_C > \rho_D$ holds when $\varepsilon < \varepsilon^*$. It means cooperation is favored in the structured populations as long as the cost to relay a message is sufficiently small. This finding on graph structures is in sharp contrast to the case of well-mixed population. Recall that defection is always favored in the latter case.

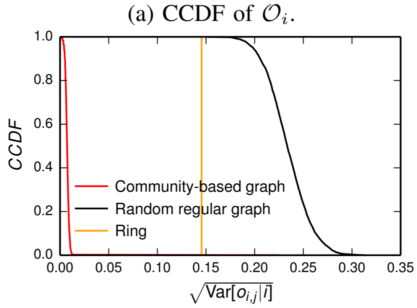
Comparing the simulation results with the critical value $\hat{\varepsilon}^* = \epsilon_{\lambda\tau,K} \cdot \frac{1}{K^2}$ predicted by equation (7), we see that $\hat{\varepsilon}^*$ agrees with the actual zero point ε^* of the simulated curves very well for all the three graph structures, although ε^* seems to deviate from $\hat{\varepsilon}^*$ (to the left) for large $\lambda\tau$, i.e. when mobile nodes encounter each other more frequently. Moreover, the theoretical $\hat{\varepsilon}^*$ slightly overestimates the actual zero point in community-based graph. This may be partially due to the fact that in community-based graph, nodes do not have uniform degree K.

C. Assumption on Destination Reachability

As mentioned in Section III-B, the DMG game assumes that every source node as well as its cooperative neighbors



(a) CCDF of $\mathcal{O}_i$.



(b) CCDF of $\sqrt{\mathrm{Var}[o_{i,j}|i]}$.

Figure 4: Neighborhood similarity statistics in graph structures. $N = 300, K = 60$. *The distribution for random regular graph and community-based graph correspond to statistics collected from 10 instances.*

are connected to the destination. The edge between source and destination allows direct message delivery, and the other edges allow the cooperative neighbors to contribute as relays. Obviously, this assumption is violated if the destination itself is a cooperative neighbor to the source. However, the effect can be negligible when the number of cooperative neighbors is large. A more critical constraint imposed on the graph structure is to make sure the cooperative neighbors, at least with high probability, are connected to the destination. A strong condition for this to be true is that every node together with its K neighbors form a $(K+1)$-clique. This can only happen when $K = N - 1$, or the network can be partitioned into disconnected $(K+1)$-cliques. In either case, the network becomes a complete graph (or several complete subgraphs), i.e. finite well-mixed population structure. Therefore we relax the constraint. For every node i, consider the metric

$$\mathcal{O}_i = \max_{j \in K_i} o_{i,j},$$
$$o_{i,j} = \frac{|K_j \cap K_i|}{\overline{K}},$$

where K_i is the set of neighbors of node i, $|\cdot|$ the size of set, and $\overline{K}$ the average node degree. Choosing the neighbor $j \in K_i$ such that $\mathcal{O}_i = o_{i,j}$ as the destination, source node i maximizes its payoff provided all the neighbors are cooperative. Graph with $\mathcal{O}_i \simeq 1$ and small $\mathrm{Var}[o_{i,j}|i]$ complies more with the population structure assumed in DMG. In the following, we study the statistics of $\mathcal{O}_i$ and $o_{i,j}$ in different type of graphs.

In *ring graph*, nodes are connected in a circular manner and every node has the same degree K. Without loss of generality, we consider K is even. One can easily obtain $\mathcal{O}_i = 1 - \frac{2}{K}$ and $\frac{1}{2} - \frac{1}{K} \leq o_{i,j} \leq 1 - \frac{2}{K}$. When $K \gg 1$, $O_i \sim 1$ but the variance of $o_{i,j}$ is large. In *random regular graph* with degree K, nodes are connected randomly. Therefore $\mathcal{O}_i$ is small as there is little overlapping between adjacent nodes' neighborhood. Finally, we observe that *community-based graph* has high $\mathcal{O}_i$ and small $\mathrm{Var}[o_{i,j}|i]$. This is not surprising since community-based graph can be regarded as several quasi-cliques linked by sparse random connections. Therefore adjacent nodes tend to have very similar neighborhood.

In Fig. 4, we plot the CCDF of $\mathcal{O}_i$ and the standard deviation $\sqrt{\mathrm{Var}[o_{i,j}|i]}$ over the three population structures. Random regular graph no doubt violates the assumption on destination reachability, as $\mathbb{P}(\mathcal{O}_i > 0.3) \simeq 0.2$, i.e. similarity in neighbors' neighborhood is too small. Therefore, the payoff function needs to be modified to reflect the relay-destination disconnection, and we expect to see a different result from Fig. 3. For ring graph, if the source-destination pairs are carefully chosen, then our assumption can be satisfied. Otherwise, due to the large standard deviation, this assumption does not hold. Lastly, community-based graph satisfies the destination reachability constraint very well, as $\mathbb{P}(\mathcal{O}_i > 0.9) \simeq 0.9$ and $\mathbb{P}(\sqrt{\mathrm{Var}[o_{i,j}|i]} > 0.02) = 0$. Therefore Fig. 3 reflects convincing results.

D. Evolution with Random Initial State

So far we have been focusing on the fixation probability of cooperators and defectors in different structured populations. In real mobile networks like DTN, the initial state is unlikely to be a single cooperator or defector, but a mix of the two strategies. Here, we repeat the simulation with a different initial condition. Instead of randomly picking one cooperator or defector, we initialize everyone's strategy randomly and independently, i.e. for each user, with probability 0.5 it is a cooperator. To faithfully report the evolution of population state in DMG, we plot the fraction of cooperators in the population at different time stages (rounds) during the simulation. Fig. 5 shows the dynamics over 1000 repeated runs. Two instances (out of 1000) of evolution dynamics on community-based graph are plotted, one towards complete cooperation (in green) and the other towards complete defection (in red). We also calculate the average cooperation rate at each time stage, and plot the averaged evolution dynamics. We see that after 50000 rounds of evolutions, the average population state is $p_C = 0.8$. It means that simulations that reach complete cooperation are the majority. This again proves that structured populations promote the evolution of cooperation.

VI. Conclusion

In this paper, we employ the EGT framework to study cooperative behavior in two-hop DTN routing game (DMG). In infinite and well-mixed populations, we show that the ESS of DMG is defection, which means without incentive schemes, cooperators become extinct in the long-term evolution. Then we investigate the promotion of cooperation in finite and structured populations. Applying the σ-rule to DMG, we analytically prove a sufficient condition under which cooperation is favored over defection on K-regular graph. This

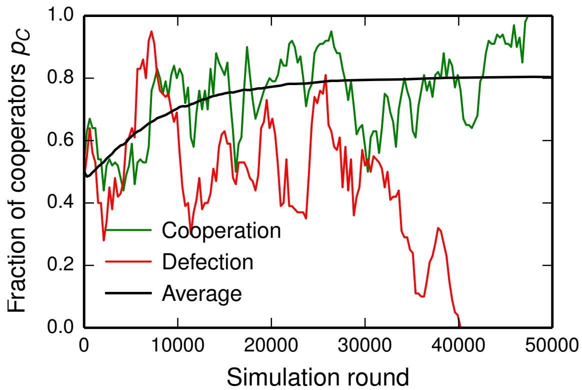


Figure 5: Evolution of cooperation rate on community-based graph with $\lambda\tau = 0.5, \omega = 0.8, N = 100, K = 10, \varepsilon = 0.2 * \hat{\varepsilon}^*$. *Simulation is repeated for 1000 times, and each simulation run has 50000 rounds of strategy updates.*

theoretical finding is verified through simulations on three different graph structures. In particular, community-based graph reflects realistic DTN mobility patterns and complies with the connectivity assumption in DMG. Most importantly, the evolution of DMG on community-based graph confirms the prevalence of cooperators even without incentives.

In future, we are interested to see if the integration of EGT model and light-weighted incentive schemes such as tit-for-tat (TFT) would further enhance cooperation in DTN routing games. In the defined DTN multi-player game, constraints such as homogeneous ICT and degree, static connectivity and the assumption of destination reachability may also be relaxed.

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Appendix A
Proof of Theorem 4

First consider the following structure coefficients σ_k:

1) Case $k = 0$:

$$\upsilon_{0,0,K} = \varsigma_{K-1,0,K} = \frac{1}{(K-1)^{K-1}},$$

and $\upsilon_{l,0,K} = \varsigma_{l,0,K} = 0$ elsewhere. Therefore

$$\begin{aligned}\sigma_0 &= \frac{(K-2)^{K-1}}{(K+2)(K+1)K^2}\frac{K^3 + [2K + (K-2)(K-1)]}{(K-1)^{K-1}} \\ &= \frac{(K-2)^{K-1}}{(K-1)^{K-1}}[\frac{K}{(K+2)(K+1)} + \frac{[2K + (K-2)(K-1)]}{(K+2)(K+1)K^2}] \\ &= (1 - \frac{1}{K-1})^{K-1}[\frac{K}{(K+2)(K+1)} + \frac{[2K + (K-2)(K-1)]}{(K+2)(K+1)K^2}] \\ &\sim \frac{1}{e}\frac{1}{K} + O(\frac{1}{K^2}) \sim \frac{1}{e}\frac{1}{K}\end{aligned} \tag{8}$$

2) Case $k = 1$:

$$\begin{aligned}&\upsilon_{0,1,K} = \upsilon_{1,1,K} = \varsigma_{K-2,1,K} = \varsigma_{K-1,1,K} = \tfrac{1}{(K-1)^{K-2}}, \\ &\upsilon_{K-1,1,K} = \varsigma_{0,1,K} = \tfrac{K-2}{(K-1)^{K-2}},\end{aligned}$$

and $\upsilon_{l,1,K} = \varsigma_{l,1,K} = 0$ elsewhere. Therefore

$$\begin{aligned}\sigma_1 = &\frac{(K-2)^{K-2}}{(K+2)(K+1)K^2}\{\frac{K^3 + (K-1)[K^2 - (K-2)]}{(K-1)^{K-2}} \\ &+ \frac{2[2K + (K-2)^2] + [2K + (K-2)(K-1)]}{(K-1)^{K-2}} \\ &+ \frac{2K^2(K-2) + [K^2 - (K-2)(K-1)](K-2)}{(K-1)^{K-1}}\} \\ &\sim \frac{2}{e}\frac{1}{K} + O(\frac{1}{K^2}) \sim \frac{2}{e}\frac{1}{K}\end{aligned} \tag{9}$$

3) Case $k = K-1$:

$$\begin{aligned}&\upsilon_{l,K-1,K} = \tfrac{1}{(K-1)^{K-1-l}} + l\tfrac{K-2}{(K-1)^l}, \\ &\varsigma_{l,K-1,K} = (K-1-l)\tfrac{K-2}{(K-1)^{K-1-l}} + \tfrac{1}{(K-1)^l},\end{aligned}$$

$$\begin{aligned}\sigma_{K-1} = &\frac{1}{(K+2)(K+1)K^2}\sum_{l=0}^{K-1}(K-l)\{[K^2 - (K-2)l]\upsilon_{l,K-1,K} \\ &+ [2K + (K-2)l]\varsigma_{l,K-1,K}\} \\ \sim &\frac{1}{\underline{K^4}}\{\frac{K^3}{(K-1)^{K-1}} + \underline{(K-1)}[\underline{K^2} - (K-2)][\frac{1}{(K-1)^{K-2}} + \underline{\frac{K-2}{K-1}}] \\ &+ O(K^2)\} + \frac{1}{K^4}\{2K^2[\frac{(K-1)(K-2)}{(K-1)^{K-1}} + 1] + ... \\ &+ 2[2K + (K-2)^2][\frac{K-2}{K-1} + \frac{1}{(K-1)^{K-2}}] \\ &+ [2K + (K-2)(K-1)]\frac{1}{(K-1)^{K-1}}\} \sim \frac{1}{K}\ [4]\end{aligned} \tag{10}$$

[4]The underlined terms yield the limitation of $\frac{1}{K}$.

4) Case $k = K$:

$$\begin{aligned}&\upsilon_{l,K,K} = \tfrac{K-2}{(K-1)^l}, \\ &\varsigma_{l,K,K} = \tfrac{K-2}{(K-1)^{K-1-l}},\end{aligned}$$

$$\begin{aligned}\sigma_K = &\frac{1}{(K+2)(K+1)K^2}\sum_{l=0}^{K-1}(K-l)\{\frac{[K^2 - (K-2)l]}{(K-1)^l} \\ &+ \frac{[2K + (K-2)l]}{(K-1)^{K-1-l}}\} \sim \frac{1}{K}.\end{aligned} \tag{11}$$

Now consider the condition (4)

$$\begin{aligned}&\sum_{k=0}^{K}\sigma_k d_k = \sum_{k=0}^{K}\sigma_k[f_{K,k}(C) - f_{K,K-k}(D)] \\ &= \sum_{k=0}^{K}\sigma_k[f_{K,k}(D) - f_{K,K-k}(D)] - K\varepsilon \\ &\overset{(a)}{=} \sum_{k=K^*+1}^{K}\sigma_k[f_{K,k}(D) - f_{K,K-k}(D)] \\ &\qquad + \sum_{k=0}^{K^*}\sigma_k[f_{K,k}(D) - f_{K,K-k}(D)] - K\varepsilon \\ &\overset{(b)}{=} \sum_{k=K^*+1}^{K}(\sigma_k - \sigma_{K-k})[f_{K,k}(D) - f_{K,K-k}(D)] - K\varepsilon \\ &\overset{(c)}{\geq} (\sigma_K - \sigma_0)[f_{K,K}(D) - f_{K,0}(D)] \\ &\qquad + (\sigma_{K-1} - \sigma_1)[f_{K,K-1}(D) - f_{K,1}(D)] - K\varepsilon,\end{aligned}$$

where in equation (a) we split the summation at $K^* = \lfloor \frac{K}{2} \rfloor$, (b) is due to the fact that $b_k = -b_{K-k}$ where $b_k = f_{K,k}(D) - f_{K,K-k}(D)$, and (c) is due to $b_k \geq 0$ and $\sigma_k - \sigma_{K-k} \geq 0$ when $K^* + 1 \leq k \leq K$.

Replacing the coefficients with values obtained from equation (8)-(11), we have

$$\begin{aligned}\sum_{k=0}^{K}\sigma_k d_k \geq &\frac{1}{K}e^{-\lambda\tau}\{(1 - \frac{1}{e})[1 - \left(\frac{1+\lambda\tau}{e^{\lambda\tau}}\right)^K] \\ &+ (1 - \frac{2}{e})[\frac{1+\lambda\tau}{e^{\lambda\tau}} - \left(\frac{1+\lambda\tau}{e^{\lambda\tau}}\right)^{K-1}]\} - K\varepsilon\end{aligned} \tag{12}$$

Therefore a sufficient condition for $\sum_{k=0}^{K}\sigma_k d_k > 0$ is

$$\begin{aligned}\varepsilon < &\frac{1}{K^2}e^{-\lambda\tau}\{(1 - \frac{1}{e})[1 - \left(\frac{1+\lambda\tau}{e^{\lambda\tau}}\right)^K] \\ &+ (1 - \frac{2}{e})[\frac{1+\lambda\tau}{e^{\lambda\tau}} - \left(\frac{1+\lambda\tau}{e^{\lambda\tau}}\right)^{K-1}]\} \sim O(\frac{1}{K^2}).\end{aligned} \tag{13}$$